Generic chiral superfield model on nonanticommutative $\mathcal{N} = \frac{1}{2}$ superspace

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Abstract

We consider the generic nonanticommutative model of chiral-antichiral superfields on $\mathcal{N} = \frac{1}{2}$ superspace. The model is formulated in terms of an arbitrary Kählerian potential, chiral and antichiral superpotentials and can include the nonanticommutative supersymmetric sigma-model as a partial case. We study a component structure of the model and derive the component Lagrangian in an explicit form with all auxiliary fields contributions. We show that the infinite series in the classical action for generic nonanticommutative model of chiral-antichiral superfields in $D = 4$ dimensions can be resumed in a compact expression which can be written as a deformation of standard Zumino's lagrangian and chiral superpotential. Problem of eliminating the auxiliary fields in the generic model is discussed and the first perturbative correction to the effective scalar potential is obtained.

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Supersymmetric field theories on deformed superspaces with nonanticommuting coordinates possess the interesting properties in classical and quantum domains. Remarkable class of such theories based on special deformation of $\mathcal{N}=1$ supersymmetry was proposed by Seiberg [1]. Seiberg's type of superspace deformation introduces the nonanticommutativity both even and odd coordinates but preserves anticommutativity in the chiral sector. As a result, the corresponding deformed superspace breaks the supersymmetry in the antichiral sector and therefore it is called $\mathcal{N}=\frac{1}{2}$ superspace. Formulation of analogous deformation in $\mathcal{N}=2$, $D=4$ superspace was given in [2]. Studying of various aspects of $\mathcal{N}=\frac{1}{2}$ supersymmetric theories has been carried out in a number of recent papers (see e.g [3], [4], [5] for $D=4$ models and [6], [7] for $D=2$ models). It is worth pointing out that the most general deformation of $\mathcal{N}=1$ superspace was constructed in work [8].

To interpret the $\mathcal{N}=\frac{1}{2}$ supersymmetric theories as conventional field models and to clarify their dynamics it is necessary to rewrite such superfield theories in the component form. Finding the component structure of the nonanticommutative theories is a highly nontrivial technical problem because of the very complicated superspace structure and therefore it demands a special study. Component form of actions for nonanticommutative theories in addition to standard terms always will contain the terms dependent on the superspace deformation parameter. Since a half of supersymmetries is broken down a symmetry between chiral and antichiral superspace coordinates is absent and some component fields can enter in the action in very cumbersome combinations.

In the papers [1], the component structure of $D=4$, $\mathcal{N}=\frac{1}{2}$ supersymmetric models Yang-Mills theory and the Wess-Zumino was studied. For this case it was shown that the deformed theory is renormalizable [4], [5] in spite of the presence of higher dimensional terms in the Lagrangian and preserves locality. However the generic $D=4$, $\mathcal{N}=\frac{1}{2}$ supersymmetric chiral-antichiral theories in four dimensions, which formulated in terms of an arbitrary Kählerian potential $K(\bar{\Phi},\Phi)$ and arbitrary chiral and antichiral superpotentials $W(\Phi)$, $\bar{W}(\bar{\Phi})$, have not been considered in the literature[1]. We will call such theories the generic chiral superfield models. It is worth pointing out that just generic chiral superfield models (with $\mathcal{N}=1$ supersymmetry) emerges in the low-energy limit of the superstring theory (see e.g. [9]) and widely used in the superstring phenomenology (see e.g. [10]).

In this paper we study the $D=4$ generic chiral superfield model in $\mathcal{N}=\frac{1}{2}$ superspace and derive its component structure. Explicit expressions for the Kählerian, chiral and antichiral superpotentials are not fixed. We show that the component action is represented as an infinite

[1] Though there is a well known connection between $D=4$, $\mathcal{N}=1$ and $D=2$, $\mathcal{N}=2$ superspaces (for recent review see Refs. [6],[7] and reference therein) a distinct feature appears in two dimensional models in comparison to four dimensional models. Such theories in two dimensions appear as a result of nontrivial reduction from $D=4$ chiral and vector multiplet. Particularly in addition to the chiral multiplet it is also possible twisted multiplets. Two-dimensional models have additional remarkable feature (like conformal and mirror symmetries) and other special properties. Therefore one can expect that the various reductions from $D=4$ nonanticommutative generic chiral-antichiral superfield model to $D=2$ will lead to various $D=2$ models and structure of $D=4$ model can not be restored on the base of the known structure of $D=2$ model. This circumstance justifies necessity for independent study of $D=4$ nonanticommutative models.

series in nonanticommutativity parameter with coefficients depending on the derivatives of above potentials. Despite this fact, it is possible to write down the action in a closed form via smoothing integrals of the Kählerian K and chiral W superpotential around the bosonic component of the chiral superfield Φ on a scale dependent on the deformation parameter and the auxiliary field $\sqrt{\det C}F$. This effect is in an agreement with an observations of Ref. [14] for $D = 2$. For $\mathcal{N} = 2$ sigma model nonanticommutativity induces simple deformations of the Zumino Lagrangian along with the holomorphic superpotential. This phenomena is interpreted as a fuzziness in target space controlled by the vacuum expectation value of the auxiliary field.

For $D = 4$ sigma-model with a pure Kähler potential we demonstrate that like in $D = 2$ sigma-model [15] one can redefine the target space metric by such a way that the symmetry between the the holomorphic and anti-holomorphic terms in the action became formally restored. We also discuss a problem of eliminating the auxiliary fields for the case of constant physical fields. We think this allows to examinate classical structure of vacua for the deformed theory.

We begin with consideration of the $\mathcal{N} = \frac{1}{2}$ deformed superspace. According to Seiberg, the coordinates of this superspace are defined such a way that Grassmannian coordinates are not complex conjugate to one another ($(\theta^\alpha)^* \neq \bar{\theta}^{\dot{\alpha}}$) and the anticommuting coordinates θ form a Clifford algebra

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} \ , \tag{1}$$

where $C^{\alpha\beta} = C^{\beta\alpha}$ is a symmetrical constant matrix. The other commutation relations are determined by the consistency of the algebra:

$$[x^m,\, \theta^\alpha] = iC^{\alpha\beta}\sigma^m_{\beta\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} \ , \quad [x^m,\, x^n] = \bar{\theta}\bar{\theta}\, C^{mn} \ , \quad \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0 \ , \tag{2}$$

where $C^{mn} = C^{\alpha\beta}\varepsilon_{\beta\gamma}\sigma^{mn}{}_\alpha{}^\gamma$, $\bar{\theta}\bar{\theta} = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} = 2\bar{\theta}^2$. In contrast to the spacetime coordinates x^m, the chiral coordinates $y^m = x^m + \frac{i}{2}\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$ can be taken commuting, while the antichiral coordinates $\bar{y}^m = y^m - i\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$ do not commute. Because the anticommutation relation of $\bar{\theta}$ remains undeformed, $\bar{\theta}$ is not the complex conjugate of θ, that is possible only in the Euclidean space. Covariant derivatives and supercharges are defined by the standard expressions

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\bar{\theta}^{\dot{\alpha}}\sigma^m_{\alpha\dot{\alpha}}\frac{\partial}{\partial y^m} \ , \quad \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \ , \tag{3}$$

$$Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} \ , \quad \bar{Q}_{\dot{\alpha}} = i\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + \theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\frac{\partial}{\partial y^m} \ , \tag{4}$$

The generators Q_α induce the transformations corresponding to unbroken $\mathcal{N} = \frac{1}{2}$ supersymmetry.

Because of the deformation (1) functions of θ must be ordered. The simplest possible ordering is the Weyl type[2] $: \theta^\alpha\theta^\beta := \frac{1}{2}[\theta^\alpha,\, \theta^\beta] = -\varepsilon^{\alpha\beta}\theta^2$. When functions of θ are multiplied,

[2] In used notations $\psi^2 = \frac{1}{2}\psi^\alpha\psi_\alpha$.

the result should be reordered again. Reordering is implementing by a noncommutative $\star$-product which is defined as follows

$$\Phi \star \Psi = \Phi \, \mathrm{e}^{-C^{\alpha\beta}\overleftarrow{Q_\alpha}\overrightarrow{Q_\beta}} \, \Psi = \Phi \left(1 - C^{\alpha\beta}\overleftarrow{Q_\alpha}\overrightarrow{Q_\beta} + \lambda \overleftarrow{Q}^2 \overrightarrow{Q}^2 \right) \Psi \ , \tag{5}$$

where $\lambda = -\frac{1}{2} C^{\alpha\beta} C_{\alpha\beta}$. The star-product (5) is invariant under the action of Q_α but is not invariant under action of $\bar{Q}$.

Described deformation preserves the half of $\mathcal{N}=1$ supersymmetry and has interesting properties in the field theory viewpoint. Replacing all ordinary products with the above $\star$-product, one can proceed studying a supersymmetric field theory in this nonanticommuting superspace taking into account that this deformed supersymmetry algebra admits well-defined representations. Namely, the chiral superfield Φ is defined by the standard relation $\bar{D}_{\dot{\alpha}} \Phi = 0$, which means $\Phi(y,\theta) = \phi(y) + \theta\kappa(y) - \theta^2 F(y)$. Antichiral superfield is also defined by the standard relation $D_\alpha \bar{\Phi} = 0$, which means $\bar{\Phi}(\bar{y},\bar{\theta}) = \bar{\phi}(\bar{y}) + \bar{\theta}\bar{\kappa}(\bar{y}) - \bar{\theta}^2 \bar{F}(\bar{y})$. As it has been demonstrated in Ref. [1], the $\star$-product (5) of the chiral superfields is again a chiral superfield; likewise, the $\star$-product of the antichiral superfields is again an antichiral superfield. This observation allows to extend well-studied anticommutative theories on nonanticommutative versions by simple replacement the point product with the star product.

The action of the generic chiral superfield model on $\mathcal{N}=1/2$ superspace is [3]

$$S_\star[\bar{\Phi},\Phi] = \int d^4x d^4\theta \, K(\bar{\Phi},\Phi)_\star + \int d^4x d^2\theta \, W(\Phi)_\star + \int d^4x d^2\bar{\theta} \, \bar{W}(\bar{\Phi})_\star \ , \tag{6}$$

where $K(\bar{\Phi},\Phi), W(\Phi), \bar{W}(\bar{\Phi})$ are the arbitrary Kählerian potential, chiral and antichiral superpotentials respectively and the superfield multiplication is defined in terms of the star-product (5).

Since the star-product (5) always begins with the point product $A \star B = A \cdot B + \cdots$, it is easy to understand that the action (6) can be written as a sum of the action for the general chiral superfield model on undeformed $\mathcal{N}=1$ superspace (see for example [11]) and some contributions higher dimensions resulting from deformation of the superspace. But the action is local. Further we will write the action (6) in component fields and study its structure.

We consider the chiral superpotential component form and write it as a Taylor series:

$$\int d^4x d^2\theta \, W(\Phi)_\star = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x d^2\theta \, W_n(\phi) f_\star^n \ , \tag{7}$$

here W_n are the expansion coefficients taken at the point ϕ and the superfield f is defines as

$$f = \Phi(y,\theta) - \phi(y) = \theta\kappa - \theta^2 F \tag{8}$$

and $f_\star^n = \underbrace{f \star f \star \cdots \star f}_{n}$. Our first aim is calculation of this star-product.

[3]The integration over Grassmannian coordinates in normalized as $\int d^2\theta = \frac{1}{2}\frac{\partial^2}{\partial\theta^2}$, $\int d^2\theta \, \theta^2 = -1$, $\int d^2\bar{\theta} \, \bar{\theta}^2 = -1$.

We begin with consideration of several first orders

$$
\begin{array}{rl}
f_\star^2 = & \theta\kappa\theta\kappa - (\kappa_\alpha + \theta_\alpha F)C^{\alpha\beta}(\kappa_\beta + \theta_\beta F) + \lambda F^2 = -2\theta^2\kappa^2 + \lambda F^2 \ , \\
f_\star^3 = & \lambda F^2 \cdot f(\theta) + 2\lambda F\kappa^2 \ , \\
f_\star^4 = & -4\theta^2\kappa^2\lambda F^2 + \lambda^2 F^4 \ , \\
f_\star^5 = & \lambda^2 F^4 \cdot f + 4\lambda^2 F^3\kappa^2 \ , \\
f_\star^6 = & -6\theta^2\kappa^2\lambda^2 F^4 + \lambda^3 F^6 \ , \\
f_\star^7 = & \lambda^3 F^6 \cdot f + 6\lambda^3 F^5 \kappa^2 \ , \\
\ldots & \ldots \ ,
\end{array}
\tag{9}
$$

Then using induction ones arrive at the following expression for even orders $2m$

$$
f_\star^{2m} = -2m\theta^2\kappa^2(\lambda F^2)^{m-1} + (\lambda F^2)^m \ , \tag{10}
$$

and for odd orders $2m+1$

$$
f_\star^{2m+1} = (\lambda F^2)^m f(\theta) + 2m\kappa^2(\lambda^m F^{2m-1}) \ . \tag{11}
$$

Collecting from (10, 11) terms with θ^2, which will survive after integration over chiral coordinates we obtain the component form of the chiral superpotential

$$
\begin{array}{rl}
\int d^4x d^2\theta \, W(\Phi)_\star = & \int d^4x \sum\limits_{n=0}^{\infty} \frac{1}{(2n)!} W_{2n}(\phi) \cdot 2n\kappa^2(\lambda F^2)^{n-1} \\
+ & \int d^4x \sum\limits_{n=0}^{\infty} \frac{1}{(2n+1)!} W_{2n+1}(\phi) \cdot \lambda^n F^{2n+1} \ .
\end{array}
\tag{12}
$$

The antichiral superpotential expansion around the scalar field $\bar\phi$ is defined as a series

$$
\int d^4x d^2\bar\theta \, \bar W_\star(\bar\Phi(\bar y, \bar\theta)) = \sum_{\bar n=0}^{\infty} \frac{1}{\bar n!} \int d^4x d^2\bar\theta \, \bar W_{\bar n}(\bar\phi) \bar f_\star^{\bar n} \ , \tag{13}
$$

here $\bar f_\star^n = \underbrace{\bar f \star \bar f \star \ldots \star \bar f}_{n}$ is a product which will be obtained below and $\bar W_{\bar n} = \frac{\partial^n \bar W(\bar\Phi)}{\partial \bar\Phi^n}$ – expansion coefficients.

The antichiral superfield being written in chiral coordinates looks as follows

$$
\bar\Phi(\bar y, \bar\theta) = \bar\Phi(y, \bar\theta) - i\theta^\alpha(\partial_{\alpha\dot\alpha}\bar\Phi(y,\bar\theta))\bar\theta^{\dot\alpha} + \theta^2\bar\theta^2\Box\bar\Phi(y,\bar\theta) \ , \tag{14}
$$

where $\Box = \frac{1}{2}\partial^{\alpha\dot\alpha}\partial_{\alpha\dot\alpha}$. That means

$$
\begin{array}{rl}
\bar f = & \bar\Phi(\bar y, \bar\theta) - \bar\phi(y) \\
= & \bar\theta^{\dot\alpha}\bar\kappa_{\dot\alpha}(y) - \bar\theta^2\bar F(y) - i\theta^\alpha(\partial_{\alpha\dot\alpha}\bar\phi(y))\bar\theta^{\dot\alpha} + i\theta^\alpha\partial_{\alpha\dot\alpha}\bar\kappa^{\dot\alpha}(y)\bar\theta^2 + \theta^2\bar\theta^2\Box\bar\phi(y) \ .
\end{array}
\tag{15}
$$

Taking into account further integration over chiral coordinates $d^2\bar\theta$ we will consider only components proportional to $\bar\theta^2$

$$
\begin{array}{l}
\bar f|_{\bar\theta^2} = -\bar F + i\theta^\alpha\partial_{\alpha\dot\alpha}\bar\kappa^{\dot\alpha} + \theta^2\Box\bar\phi \ , \\
\bar f_\star^2|_{\bar\theta^2} = -2\bar\kappa^2 + 2i\theta^\alpha(\partial_{\alpha\dot\alpha}\bar\phi)\bar\kappa^{\dot\alpha} + \theta^2\partial^{\alpha\dot\alpha}\bar\phi\partial_{\alpha\dot\alpha}\bar\phi + C^{\alpha\beta}\partial_\alpha^{\dot\alpha}\bar\phi\partial_{\beta\dot\alpha}\bar\phi \ .
\end{array}
\tag{16}
$$

The last term in the second line is equal to zero due to a property $\partial^{\dot\alpha}_{(\alpha}\bar\phi\partial_{\beta)\dot\alpha}\bar\phi = -\partial_{\dot\alpha(\alpha}\bar\phi\partial^{\dot\alpha}_{\beta)}\bar\phi \equiv 0$ and can be dropped. Therefore deformation doesn't affect on antichiral sector $\bar f \star \bar f = \bar f \cdot \bar f$ in accordance with Ref. [1]. All other orders is equal to zero, because each $\bar f$ contains $\bar\theta$ and $\bar f^2 \sim \bar\theta^2$, i.e. $\bar f^n_\star = 0,\ n > 2$. Rewriting Eq.(13) as an integral over the whole superspace

$$\int d^4x d^2\bar\theta\, \bar W_\star = -\int d^4x d^2\bar\theta d^2\theta\theta^2 \bar W_\star(\bar\phi + \bar f) \ , \tag{17}$$

where $\bar\phi = \bar\phi(y)$ and using the property $-\int d^2\bar\theta d^2\theta\theta^2\bar W(\bar\phi(x)) = 0$ one can finally write a component form for the antichiral superpotential

$$\begin{aligned}\int d^4x d^2\bar\theta\, \bar W_\star = & -\int d^2\bar\theta d^2\theta\theta^2\bar\theta^2 \left(-\bar W_{\bar 1}(\bar\phi)\bar F + \frac{1}{2}\bar W_{\bar 2}(\bar\phi)(-2\bar\kappa^2)\right) \\ & = \int d^4x \left(\bar W_{\bar 1}(\bar\phi)\bar F + \bar W_{\bar 2}(\bar\phi)\bar\kappa^2\right) \ ,\end{aligned} \tag{18}$$

where the expansion coefficients $\bar W_{\bar 1}, \bar W_{\bar 2}$ were defined above. As one can see the great difference between forms of chiral and antichiral superpotentials appears. Obviously the action doesn't have Hermiticity properties.

The most nontrivial calculation is related to the Kähler potential decomposition. We will suppose that its expansion is fully symmetrical in powers of $\bar f$ and f, i.e.

$$K(\Phi,\bar\Phi)_\star = \sum_{m=0}^{\infty}\frac{1}{m!}\left(f\frac{\partial}{\partial\Phi} + \bar f\frac{\partial}{\partial\bar\Phi}\right)^m_\star K(\Phi,\bar\Phi)|_{\Phi=\phi,\ \bar\Phi=\bar\phi} \ , \tag{19}$$

Such kind of ordering leads to the following expansion

$$\begin{aligned}K(\Phi,\bar\Phi)_\star = \ & K(\phi,\bar\phi) + K_1 f + K_{\bar 1}\bar f \\ + \ & \tfrac{1}{2}K_2 f\star f + \tfrac{1}{2}K_{\bar 2}\bar f\star\bar f + \tfrac{1}{2}K_{1\bar 1}(f\star\bar f + \bar f\star f) \\ + \ & \tfrac{1}{3!}K_3 f\star f\star f \\ + \ & \tfrac{1}{3!}K_{2\bar 1}(f\star f\star\bar f + f\star\bar f\star f + \bar f\star f\star f) \\ + \ & \tfrac{1}{3!}K_{1\bar 2}(f\star\bar f\star\bar f + \bar f\star\bar f\star f + \bar f\star f\star\bar f) + \ldots \\ = \ & \sum_n K_n f^n_\star + \sum_{\bar n} K_{\bar n}\bar f^{\bar n}_\star + \sum_{n,\bar n} K_{n\bar n}[f^n_\star \star \bar f^{\bar n}_\star] \ ,\end{aligned} \tag{20}$$

where $[\bar f^{\bar n}\star f^n]$ is a fully symmetrized star-product including all possible permutations. For instance, $[f\star f\star\bar f] = f\star f\star\bar f + f\star\bar f\star f + \bar f\star f\star f$. From (9) follows that unmixed products like $f^n_\star$ for any n will not give contribution to the Kähler potential because they do not contain factor $\bar\theta^2$ we need for further integration over $\int d^2\bar\theta$. Unmixed star products $\bar f^{\bar n}_\star$ for $n = 3$ and higher will vanish and hence, do not contribute to the action. Thus, we should study the star product $[\bar f^{\bar n}\star f^m]$ of arbitrary integer m with $\bar n = 1, 2$. Indeed, let us consider the possible mixed star-products in the expansion (20). Using the superfields f and $\bar f$ (8, 15), we obtain for the first order mixed product the following expression

$$\begin{aligned}f\star\bar f = \theta\kappa(\bar\theta\bar\kappa - \bar\theta^2\bar F - i\theta^\alpha\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha}\bar\phi + i\theta^\alpha\partial_{\alpha\dot\alpha}\bar\kappa^{\dot\alpha}\bar\theta^2) - \theta^2 F(\bar\theta\bar\kappa - \bar\theta^2\bar F) - \lambda F\bar\theta^2\Box\bar\phi \\ -C^{\alpha\beta}\kappa_\alpha(-i\partial_{\beta\dot\alpha}\bar\theta^{\dot\alpha}\bar\phi + i\partial_{\beta\dot\alpha}\bar\kappa^{\dot\alpha}\bar\theta^2 + \theta_\beta\bar\theta^2\Box\bar\phi) - C^{\alpha\beta}F\theta_\alpha(i\partial_{\beta\dot\alpha}\bar\theta^{\dot\alpha}\bar\phi - i\partial_{\beta\dot\alpha}\bar\kappa^{\dot\alpha}\bar\theta^2) \ .\end{aligned} \tag{21}$$

After symmetrization most of the terms here will disappear and ones get

$$\frac{1}{2}[f\star\bar f+\bar f\star f]=\theta\kappa(\bar\theta\bar\kappa-\bar\theta^2\bar F-i\theta^\alpha\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha}\bar\phi+i\theta^\alpha\partial_{\alpha\dot\alpha}\bar\kappa^{\dot\alpha}\bar\theta^2)-\theta^2F(\bar\theta\bar\kappa-\bar\theta^2\bar F)-\lambda F\bar\theta^2\Box\bar\phi\ . \quad (22)$$

Further, because we are interested in terms with coefficient $\bar\theta^2$, we concentrate only on these terms. Third order mixed star-product looks like

$$\frac{1}{3!}[f\star f\star\bar f+f\star\bar f\star f+\bar f\star f\star f]|_{\theta^2\bar\theta^2}=\kappa^2\bar F+\frac{1}{2}\lambda F^2\Box\bar\phi\ . \quad (23)$$

For obtaining the other products ones use the property [6]

$$\frac{1}{(n+1)!}[\bar f\star f^n]\,|_{\theta^2\bar\theta^2}=\frac{1}{(n)!}\bar f\star f^n|_{\theta^2\bar\theta^2}\ , \quad (24)$$

which leads to the considerable simplifications. Direct calculation gives us factors at the coefficients $K_{\bar 1n}$:

$$\begin{array}{l}\bar f\star f_\star^{2n}|_{\theta^2\bar\theta^2}=2n\kappa^2(\lambda F^2)^{n-1}\bar F+(\lambda F^2)^n\Box\bar\phi\ ,\\ \bar f\star f_\star^{(2n+1)}|_{\theta^2\bar\theta^2}=\lambda^nF^{2n+1}\bar F-i\kappa^\alpha\partial_{\alpha\dot\alpha}\bar\kappa^{\dot\alpha}\lambda^nF^{2n}+2n\kappa^2\lambda^nF^{2n-1}\Box\bar\phi\ .\end{array} \quad (25)$$

Next, we compute factors at the coefficients $K_{\bar 2n}$ by the same way

$$\begin{array}{l}f_\star^{2n}\star\bar f_\star^2|_{\theta^2\bar\theta^2}=2\kappa^2\bar\kappa^22n(\lambda F^2)^{n-1}+\lambda^nF^{2n}\partial^{\alpha\dot\alpha}\bar\phi\partial_{\alpha\dot\alpha}\bar\phi\ ,\\ f_\star^{2n+1}\star\bar f_\star^2|_{\theta^2\bar\theta^2}=-(\lambda F^2)^n2i\kappa^\alpha\bar\kappa^{\dot\alpha}(\partial_{\alpha\dot\alpha}\bar\phi)+2\bar\kappa^2\lambda^nF^{2n+1}+2n\kappa^2\lambda^nF^{2n-1}\partial^{\alpha\dot\alpha}\bar\phi\partial_{\alpha\dot\alpha}\bar\phi\ .\end{array} \quad (26)$$

Using (12, 18, 25, 26) we write the full Lagrangian in component form for the $\mathcal{N}=\frac{1}{2}$ nonanticommutative generic chiral superfield model (6) as a infinite series expansion in the parameter deformation

$$\begin{aligned}\mathcal{L}_\star=\ & K(\Phi,\bar\Phi)_\star|_{\theta^2\bar\theta^2}+W(\Phi)_\star|_{\theta^2}+\bar W(\bar\Phi)_\star|_{\bar\theta^2}\\ =\ & \bar W_{\bar 1}\bar F+\bar W_{\bar 2}\bar\kappa^2+\sum_{n=0}^\infty\frac{\lambda^nF^{2n}}{(2n+1)!}\left(W_{2n+2}\kappa^2+W_{2n+1}F\right)\\ +\ & \bar F\sum_{n=0}^\infty\frac{\lambda^nF^{2n}}{(2n+1)!}\left(K_{\bar 1(2n+2)}\kappa^2+K_{\bar 1(2n+1)}F\right)\\ +\ & \Box\bar\phi\sum_{n=0}^\infty\frac{\lambda^nF^{2n-1}}{(2n)!}\left(\frac{2n}{2n+1}K_{\bar 1(2n+1)}\kappa^2+K_{\bar 1(2n)}F\right)\\ +\ & \sum_{n=0}^\infty\frac{\lambda^nF^{2n}}{(2n+1)!}K_{\bar 1(2n+1)}(i\kappa^\alpha\partial_\alpha^{\dot\alpha}\bar\kappa_{\dot\alpha})\\ +\ & \frac{1}{2}\partial^{\alpha\dot\alpha}\bar\phi\partial_{\alpha\dot\alpha}\bar\phi\sum_{n=0}^\infty\frac{\lambda^nF^{2n-1}}{(2n)!}\left(K_{\bar 2(2n+1)}\frac{2n}{2n+1}\kappa^2+K_{\bar 2(2n)}F\right)\\ +\ & \sum_{n=0}^\infty\frac{\lambda^nF^{2n+1}}{(2n+1)!}K_{\bar 2(2n+1)}\bar\kappa^2\\ +\ & \sum_{n=0}^\infty\left[\frac{1}{(2n)!}K_{\bar 2(2n)}\left(2n\kappa^2\bar\kappa^2(\lambda F^2)^{n-1}\right)\right.\\ +\ & \left.\frac{1}{(2n+1)!}K_{\bar 2(2n+1)}\left((\lambda F^2)^ni\kappa^\alpha(\partial_\alpha^{\dot\alpha}\bar\phi)\bar\kappa_{\dot\alpha}\right)\right]\ ,\end{aligned} \quad (27)$$

where all coefficients are calculated at the point ϕ, i.e. $W_n = W_n(\phi)$, $\bar{W}_{\bar{n}} = \bar{W}_{\bar{n}}(\bar{\phi})$, $K_{n\bar{n}} = K_{n\bar{n}}(\phi, \bar{\phi})$. The Lagrangian (27) can be written as a sum

$$\mathcal{L}_\star = \mathcal{L} + \Delta\mathcal{L}(\lambda) \ , \tag{28}$$

here $\mathcal{L}$ is the component Lagrangian for the generic chiral superfield model in $\mathcal{N} = 1$ superspace with the action (see e.g. Ref. [11])

$$S[\bar{\Phi}, \Phi] = \int d^4x d^4\theta \, K(\bar{\Phi}, \Phi) + \int d^4x d^2\theta \, W(\Phi) + \int d^4x d^2\bar{\theta} \, \bar{W}(\bar{\Phi}) \ . \tag{29}$$

Further we will explore some properties of the model (29). In particular, being expanded around the bosonic fields $\phi, \bar{\phi}$, the component form for the Lagrangian (29) is written as

$$\begin{aligned}
\mathcal{L} = \Big(& -g\frac{1}{2}\partial^{\alpha\dot{\alpha}}\phi\partial_{\alpha\dot{\alpha}}\bar{\phi} + ig\kappa^\alpha\partial_\alpha^{\dot{\alpha}}\bar{\kappa}_{\dot{\alpha}} - K_{1\bar{2}}i\kappa^\alpha\bar{\kappa}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\bar{\phi} \\
& + gF\bar{F} + K_{2\bar{1}}\kappa^2\bar{F} + K_{1\bar{2}}\bar{\kappa}^2F + W_1F + \bar{W}_{\bar{1}}\bar{F} \\
& + W_2\kappa^2 + \bar{W}_{\bar{2}}\bar{\kappa}^2 + K_{2\bar{2}}\kappa^2\bar{\kappa}^2 \Big) \ ,
\end{aligned} \tag{30}$$

where we introduced the Kählerian metrics $g = K_{1\bar{1}}(\bar{\phi}, \phi) = \partial^2 K(\bar{\phi}, \phi)/\partial\phi\partial\bar{\phi}$. Such a form can be directly obtained from (27) as a coefficient at $n = 0$.

Equations of motion on auxiliary fields $F, \bar{F}$ in (30) have the solutions

$$F = -g^{-1}(\bar{W}_{\bar{1}} + K_{2\bar{1}}\kappa^2) \ , \quad \bar{F} = -g^{-1}(W_1 + K_{1\bar{2}}\bar{\kappa}^2) \ . \tag{31}$$

These solutions can be used to eliminate the auxiliary fields from the Lagrangian (30). It gives to

$$\begin{aligned}
\mathcal{L} = & -g\frac{1}{2}\partial^{\alpha\dot{\alpha}}\phi\partial_{\alpha\dot{\alpha}}\bar{\phi} + ig\kappa^\alpha\partial_\alpha^{\dot{\alpha}}\bar{\kappa}_{\dot{\alpha}} - K_{1\bar{2}}i\kappa^\alpha\bar{\kappa}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\bar{\phi} \\
& - g^{-1}K_{2\bar{1}}W_1\kappa^2 - g^{-1}K_{2\bar{1}}K_{1\bar{2}}\bar{\kappa}^2\kappa^2 - g^{-1}K_{1\bar{2}}\bar{W}_{\bar{1}}\bar{\kappa}^2 - g^{-1}W_1\bar{W}_{\bar{1}} \\
& + W_2\kappa^2 + \bar{W}_{\bar{2}}\bar{\kappa}^2 + K_{2\bar{2}}\kappa^2\bar{\kappa}^2 \ ,
\end{aligned} \tag{32}$$

where the first line is kinetic terms while the second and third line present the potential for the model (29)

$$\begin{aligned}
U = & \, g^{-1}K_{2\bar{1}}W_1\kappa^2 + g^{-1}K_{2\bar{1}}K_{1\bar{2}}\bar{\kappa}^2\kappa^2 + g^{-1}K_{1\bar{2}}\bar{W}_{\bar{1}}\bar{\kappa}^2 \\
& -g^{-1}W_1\bar{W}_{\bar{1}} - W_2\kappa^2 - \bar{W}_{\bar{2}}\bar{\kappa}^2 - K_{2\bar{2}}\kappa^2\bar{\kappa}^2 \ .
\end{aligned} \tag{33}$$

Since the general chiral superfield model on $\mathcal{N} = \frac{1}{2}$ superspace includes the action (29) we expect that the potential (33) for the model under consideration should be modified by dependent on deformation parameter λ terms.

The Lagrangian (30) presents all terms independent on λ (which survive at $\lambda = 0$ in (27)), while the new term $\Delta\mathcal{L}$ is conditioned by the superspace deformation and depends on

the parameter λ

$$
\begin{aligned}
\Delta\mathcal{L}(\lambda) = &\sum_{n=1}^{\infty}\frac{\lambda^n F^{2n}}{(2n+1)!}\Big(W_{2n+2}\kappa^2 + W_{2n+1}F\Big) \\
+ &\bar{F}\sum_{n=1}^{\infty}\frac{\lambda^n F^{2n}}{(2n+1)!}\Big(K_{\bar{1}(2n+2)}\kappa^2 + K_{\bar{1}(2n+1)}F\Big) \\
+ &\Box\bar{\phi}\sum_{n=1}^{\infty}\frac{\lambda^n F^{2n-1}}{(2n)!}\Big(\frac{2n}{2n+1}K_{\bar{1}(2n+1)}\kappa^2 + K_{\bar{1}(2n)}F\Big) \\
+ &\sum_{n=1}^{\infty}\frac{\lambda^n F^{2n}}{(2n+1)!}K_{\bar{1}(2n+1)}(i\kappa^\alpha\partial_\alpha^{\dot{\alpha}}\bar{\kappa}_{\dot{\alpha}}) \\
+ &\frac{1}{2}\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\sum_{n=1}^{\infty}\frac{\lambda^n F^{2n-1}}{(2n)!}\Big(K_{\bar{2}(2n+1)}\frac{2n}{2n+1}\kappa^2 + K_{\bar{2}(2n)}F\Big) \\
+ &\sum_{n=1}^{\infty}\frac{\lambda^n F^{2n+1}}{(2n+1)!}K_{\bar{2}(2n+1)}\bar{\kappa}^2 \\
+ &\sum_{n=1}^{\infty}\Bigg[\frac{1}{(2n)!}K_{\bar{2}(2n)}\Big(2n\kappa^2\bar{\kappa}^2(\lambda F^2)^{n-1}\Big) \\
+ &\frac{1}{(2n+1)!}K_{\bar{2}(2n+1)}\Big((\lambda F^2)^n i\kappa^\alpha(\partial_\alpha^{\dot{\alpha}}\bar{\phi})\bar{\kappa}_{\dot{\alpha}}\Big)\Bigg] ,
\end{aligned}
\tag{34}
$$

The relations (28, 30, 34) give the component structure for the considered deformed generic chiral superfield model.

The obtained representation for the action (27) is complicated and inaccessible even in the classical domain. Now we show that the infinite series (27) can be resummed in a compact expression similar to the standard Zumino's Lagrangian [12] with the deformed Kähler potential and the chiral superpotential plus a finite number of higher dimensional terms with field-dependent couplings. In the analogy with the trick used in the papers [13, 14] we introduce "fuzzy fields" controlled by the auxiliary fields $\phi + \tau\sqrt{\lambda}F$ on interval $-1 \le \tau \le 1$:

$$
\begin{aligned}
\mathcal{W}^{(0)}(\phi, F) &= \frac{1}{2}\int_{-1}^{1} d\tau W(\phi+\tau\xi) , \quad \xi = \sqrt{\lambda}F , \\
\mathcal{K}^{(0)}(\phi, F, \bar{\phi}) &= \frac{1}{2}\int_{-1}^{1} d\tau K(\phi+\tau\xi, \bar{\phi}) , \\
\mathcal{K}^{(1)}(\phi, F, \bar{\phi}) &= \frac{1}{2}\int_{-1}^{1} d\tau \tau K(\phi+\tau\xi, \bar{\phi}) , \\
\mathcal{K}^{(-1)}(\phi, F, \bar{\phi}) &= \frac{1}{2}\int_{-1}^{1} d\tau \frac{\partial}{\partial\tau}(\tau\cdot K(\phi+\tau\xi, \bar{\phi})) .
\end{aligned}
\tag{35}
$$

Then (27) can be rewritten in a compact form:

$$
\begin{aligned}
\mathcal{L}_\star = \bar{W}_{\bar{1}}\bar{F} + \bar{W}_{\bar{2}}\bar{\kappa}^2 + F\mathcal{W}_1^{(0)} + \kappa^2\mathcal{W}_2^{(0)} + (\bar{F}F + i\kappa^\alpha\partial_\alpha^{\dot{\alpha}}\bar{\kappa}_{\dot{\alpha}})\mathcal{K}_{1\bar{1}}^{(0)} + \kappa^2\bar{F}\mathcal{K}_{2\bar{1}}^{(0)} \\
+\Box\bar{\phi}\mathcal{K}_{\bar{1}}^{(-1)} + \sqrt{\lambda}\kappa^2\Box\bar{\phi}\mathcal{K}_{2\bar{1}}^{(1)} + \frac{1}{2}\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\mathcal{K}_{\bar{2}}^{(-1)} + \bar{\kappa}^2F\mathcal{K}_{1\bar{2}}^{(0)} \\
+i\kappa^\alpha(\partial_\alpha^{\dot{\alpha}}\bar{\phi})\bar{\kappa}_{\dot{\alpha}}\mathcal{K}_{1\bar{2}}^{(0)} + \sqrt{\lambda}\kappa^2\frac{1}{2}\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\mathcal{K}_{2\bar{2}}^{(1)} + \kappa^2\bar{\kappa}^2\mathcal{K}_{2\bar{2}}^{(0)} .
\end{aligned}
\tag{36}
$$

It is quite remarkable that the deformation encoded by new geometric quantities which look like the "metric" $\mathcal{K}^{(0)}_{1\bar{1}}$, "connection" $\mathcal{K}^{(0)}_{2\bar{1}}$ and the "curvature" $\mathcal{K}^{(0)}_{2\bar{2}}$ in the smearing target space. But there is no any certainty that this quantities are really consistent among themselves and correspond to some geometrical structure of the target space manifold.

It is easy to see that the first and the third terms in the second line of (36) up to space-time derivatives of the auxiliary field F can be written as $-\frac{1}{2}\partial^{\alpha\dot{\alpha}}\phi\partial_{\alpha\dot{\alpha}}\bar{\phi}\mathcal{K}^{(-1)}_{1\bar{1}}$. Second term in the second line plus second term in the third line is $-\sqrt{\lambda}\kappa^2\frac{1}{2}\partial^{\alpha\dot{\alpha}}\phi\partial_{\alpha\dot{\alpha}}\bar{\phi}\mathcal{K}^{(1)}_{3\bar{1}} - \sqrt{\lambda}(\partial^{\alpha\dot{\alpha}}\kappa^2)\frac{1}{2}\partial_{\alpha\dot{\alpha}}\bar{\phi}\mathcal{K}^{(1)}_{2\bar{1}}$. We can rewrite (36) in the canonical form with a proper kinetic term for the scalars $\partial^{\alpha\dot{\alpha}}\phi\partial_{\alpha\dot{\alpha}}\bar{\phi}\mathcal{K}^{(0)}_{1\bar{1}}$ but, due to the extra dependence of $\mathcal{K}^{(0)}(\phi, F, \bar{\phi})$ of the auxiliary field F, there will be new terms containing one derivative of the auxiliary field $\partial^{\alpha\dot{\alpha}}F$. At the limit $\lambda \to 0$ this terms will vanish. This is the great difference between (30) and (36).

Taking into account properties $\frac{\partial}{\partial F}\mathcal{K}^{(0)} = \sqrt{\lambda}\frac{\partial}{\partial\phi}\mathcal{K}^{(1)}$ and $\mathcal{K}^{(-1)} = \mathcal{K}^{(0)} + F\frac{\partial}{\partial F}\mathcal{K}^{(0)}$ one can note that Eq. (36) has the structure similar to Eq.(30), where the quantity $\mathcal{K}^{(0)}_{1\bar{1}}$ may be considered as a deformed metric dependent on the auxiliary field F. Combining terms one can rewrite the expression (36) via single function $\mathcal{K}^{(0)}$ as follows:

$$
\begin{aligned}
\mathcal{L}_\star = \bar{W}_{\bar{1}}\bar{F} + \bar{W}_{\bar{2}}\bar{\kappa}^2 + F\mathcal{W}^{(0)}_1 + \kappa^2\mathcal{W}^{(0)}_2 + (\bar{F}F + i\kappa^\alpha\partial^{\dot{\alpha}}_\alpha\bar{\kappa}_{\dot{\alpha}})\mathcal{K}^{(0)}_{1\bar{1}} + \kappa^2\bar{F}\mathcal{K}^{(0)}_{2\bar{1}} \\
+\Box\bar{\phi}\mathcal{K}^{(0)}_{\bar{1}} + F\Box\bar{\phi}\partial_F\mathcal{K}^{(0)}_{\bar{1}} + \kappa^2\Box\bar{\phi}\partial_F\mathcal{K}^{(0)}_{1\bar{1}} + \frac{1}{2}\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\mathcal{K}^{(0)}_{\bar{2}} + \frac{1}{2}F\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\partial_F\mathcal{K}^{(0)}_{\bar{2}} \\
+\bar{\kappa}^2F\mathcal{K}^{(0)}_{1\bar{2}} + i\kappa^\alpha(\partial^{\dot{\alpha}}_\alpha\bar{\phi})\bar{\kappa}_{\dot{\alpha}}\mathcal{K}^{(0)}_{1\bar{2}} + \kappa^2\frac{1}{2}\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\partial_F\mathcal{K}^{(0)}_{1\bar{2}} + \kappa^2\bar{\kappa}^2\mathcal{K}^{(0)}_{2\bar{2}} ,
\end{aligned}
\tag{37}
$$

where $\partial_F = \partial/\partial F$. Comparison with Eq. (30) shows the terms which spoil the Kähler structure of the Lagrangian

$$
\begin{aligned}
\mathcal{L}_\star = \mathcal{L}(W \to \mathcal{W}^{(0)},\ K \to \mathcal{K}^{(0)}) \\
+\Box\bar{\phi}F\partial_F\mathcal{K}^{(0)}_{\bar{1}} + \frac{1}{2}F\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\partial_F\mathcal{K}^{(0)}_{\bar{2}} + \kappa^2\Box\bar{\phi}\partial_F\mathcal{K}^{(0)}_{1\bar{1}} + \kappa^2\frac{1}{2}\partial^{\alpha\dot{\alpha}}\bar{\phi}\partial_{\alpha\dot{\alpha}}\bar{\phi}\partial_F\mathcal{K}^{(0)}_{1\bar{2}} .
\end{aligned}
\tag{38}
$$

This is quite natural because though the initial Lagrangian (6) has the Kählerian form the $\mathcal{N} = \frac{1}{2}$ superspace is not a Kählerian manifold ($\bar{\theta}$ is not the complex conjugate of θ). This property was firstly noted in the recent work [14] for $\mathcal{N} = 2$, $D = 2$ nonanticommutative sigma model.

Now consider generic nonanticommuting supersymmetric sigma-model (i.e. the model without superpotential W but with arbitrary Kahlerian potential K). It was shown in Ref. [15] that for $D = 2$, $\mathcal{N} = 2$ nonanticommuting sigma-model the component action infinite series can be resummed to a very simple and clear form. Let's consider such possibility for $D = 4$, $\mathcal{N} = \frac{1}{2}$ nonanticommuting sigma-model. In the linear approximation on λ the Lagrangian (27) after introducing a new metric $\tilde{g} = g + \frac{\lambda}{6}F^2K_{3\bar{1}}$ can be rewritten as follows

$$
\begin{aligned}
\mathcal{L}_\star = -\frac{1}{2}\partial^{\alpha\dot{\alpha}}\phi\partial_{\alpha\dot{\alpha}}\bar{\phi}(g + \frac{\lambda}{2}F^2K_{3\bar{1}} + \frac{\lambda}{3}F\kappa^2K_{4\bar{1}}) + (F\bar{F} + i\kappa^\alpha\partial^{\dot{\alpha}}_\alpha\bar{\kappa}_{\dot{\alpha}})\tilde{g} \\
+i\kappa^\alpha(\partial^{\dot{\alpha}}_\alpha\bar{\phi})\bar{\kappa}_{\dot{\alpha}}\tilde{g}_{\bar{1}} + \bar{F}\kappa^2\tilde{g}_1 + F\bar{\kappa}^2\tilde{g}_{\bar{1}} + \kappa^2\bar{\kappa}^2\tilde{g}_{1\bar{1}} \\
+\frac{\lambda}{6}F\kappa^2\cdot\partial^{\alpha\dot{\alpha}}(\partial_{\alpha\dot{\alpha}}\bar{\phi}K_{3\bar{1}}) + \frac{1}{4}F^2\cdot\partial^{\alpha\dot{\alpha}}(\partial_{\alpha\dot{\alpha}}\bar{\phi}K_{2\bar{1}})
\end{aligned}
\tag{39}
$$

The equation of motion for field F following from this Lagrangian is

$$F\tilde{g} + \kappa^2 \tilde{g}_1 = 0 \tag{40}$$

and at that time two last terms $\sim \kappa^4$ that vanish. This allows to note that the expression $(g + \frac{\lambda}{2}F^2 K_{3\bar{1}} + \frac{\lambda}{3}F\kappa^2 K_{4\bar{1}})$ become equal $\tilde{g}$. Thus we see that Lagrangian (39) in the first order on λ is one to one correspond to the Zumino Lagrangian with the metric $\tilde{g}$. We point out that such a consideration is true only $W = 0$ and for a singlet fermionic field. In accord with Ref. [15] one can verify that the action given by Eq. (27) at $W = 0$ and $\bar{W} = 0$ in all orders on λ can be rewritten in the form Eq. (39).

Next we discuss elimination of the auxiliary fields $F, \bar{F}$ from the component Lagrangian (27) keep in mind the task investigate the structure of classical vacua. The Lagrangian (27) is linear in $\bar{F}$ but strongly nonlinear in F. Therefore it is difficult to expect that we obtain the exact solution on F and $\bar{F}$ but we can perturbatively find several first corrections to the scalar potential and to the scalar - fermion interaction terms. In particular, the scalar potential is the most important object for study the possible vacua of the theory and a symmetry breaking mechanism studying. Let's consider only space-time independent vacuum expectation values for the scalar and fermionic physical fields. We suppose that

$$F = F_0 + F_1 + \cdots \ , \quad \bar{F} = \bar{F}_0 + \bar{F}_1 + \cdots \ , \tag{41}$$

where F_0 and $\bar{F}_0$ are the solutions for auxiliary fields equations of motion presented by the Eq. (31), $F_n \sim \lambda^n$, $\bar{F}_{\bar{n}} \sim \lambda^{\bar{n}}$ are the corrections. Substituting (41) into the Lagrangian (27) and keeping only linear in λ terms without derivatives we obtain first corrections to the auxiliary fields

$$\begin{aligned} -gF_1 = & \ \lambda\left(\tfrac{1}{6}F_0^2 K_{4\bar{1}}\kappa^2 + \tfrac{1}{2}F_0^3 K_{3\bar{1}}\right) \ , \\ -g\bar{F}_1 = & \ \lambda\bar{F}_0 F_0\left(\tfrac{1}{3}K_{4\bar{1}}\kappa^2 + \tfrac{1}{2}F_0 K_{3\bar{1}}\right) + \tfrac{1}{2}\lambda F_0^2 K_{3\bar{2}}\bar{\kappa}^2 \\ + & \ \lambda\tfrac{1}{3}F_0 K_{4\bar{2}}\kappa^2\bar{\kappa}^2 + \lambda F_0\left(\tfrac{1}{3}W_4\kappa^2 + \tfrac{1}{2}F_0 W_3\right) \ . \end{aligned} \tag{42}$$

This gives us, in addition to the ordinary potential U presented by (33), a linearly dependent on λ correction

$$\begin{aligned} \Delta U_1(\lambda) = & \ g(F_1\bar{F}_0 + \bar{F}_1 F_0) + F_1(W_1 + K_{1\bar{2}}\bar{\kappa}^2) + \bar{F}_1(\bar{W}_{\bar{1}} + K_{2\bar{1}}\kappa^2) \\ + & \ \tfrac{\lambda}{6}F_0^2 K_{4\bar{2}}\kappa^2\bar{\kappa}^2 + \tfrac{\lambda}{6}F_0^2\bar{F}_0(K_{4\bar{1}}\kappa^2 + F_0 K_{3\bar{1}}) \\ + & \ \tfrac{\lambda}{6}F_0^2(W_4\kappa^2 + F_0 W_3) + \tfrac{\lambda}{6}F_0^3 K_{3\bar{2}}\bar{\kappa}^2 \ , \end{aligned} \tag{43}$$

where $F_0, \bar{F}_0$ and $F_1, \bar{F}_1$ are given in Eqs. (31, 42) respectively. As a result, we finally get that the potential U given by Eq.(32) and a series of additional terms dependent on λ. Considering the expressions (33, 42, 43) one can see that the full potential as a function of the scalar fields and fermionic condensate $\langle\kappa^2\rangle$ can be as positive as negative defined depending on concrete forms of the Kählerian and chiral superpotentials. It means, in general, that at nonvanishing λ the potential possesses a possibility to get a minimum, though the initial potential (33) has none minimum. Therefore one can expect some kind of symmetry breaking

in the model under consideration. This very interesting aspect is out of this paper subject and deserves a separate study.

To summarize, we have considered the supersymmetric generic chiral superfield model on $\mathcal{N} = \frac{1}{2}$ nonanticommutative superspace. This model is given in terms of arbitrary Kählerian potential, chiral and antichiral superpotentials. We have developed a general procedure for deriving the component structure of the model and obtained the component action in the explicit form as a infinite series in the nonanticommutativity parameter. This series is summed up into compact expression using the specific integral representations. It was shown that the additional "deformed" part of the action allows a perturbative translation invariant solution for the auxiliary fields equations of motion. Leading corrections to nondeformed potential are calculated. The results obtained can be applied to studying a wide class of various $\mathcal{N} = \frac{1}{2}$ chiral superfield models including supersymmetric sigma-models and models with different chiral and antichiral superpotentials.

After we had put this work in the hep-th ArXiv, the work [13] appeared, where the component structure for chiral superpotential (12, 18) has been obtained. After that two papers [14] and [15] related to studying the component structure of $N = 2$, $D = 2$ nonanticommuting sigma models appeared.

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